Semantics of Programming Languages
Special Topics – CEEPUS 2022
Exam Test

Name: Email:
Date:

Write a name and date where indicated. Otherwise, the examination cannot be marked. Do not speak to any other student during the examination. If you speak to another student, your examination will be canceled. Use a pen, not a pencil. Please, write neatly and clearly.

Notes

- The exam comprises four main parts, which consist of several sub-questions. You will have 60 minutes time to answer the questions.
- Tables for natural and structural operational semantics are attached, the evaluation scale is on the page no. 5.
- Please, write down the answers to your questions in the exam booklet handed out to you.
- For drafts use the blank paper provided by the university.
- If the space in the booklet turns out to be insufficient, please use the university paper for additional answers and return them with the booklet.
- No questions will be answered during the exam. If you are not sure about interpreting a question, you may write down additional assumptions you made in order to proceed with your solution

Question 1

[12 p.] Given a program

$$\begin{array}{l} y := 0; \\ \texttt{while}\ (1 \leq x)\ \texttt{do}\ (\\ \quad y := y + 2 * x; \\ \quad x := x - 1\ ); \end{array}$$

Construct a derivation sequence of structural operational semantics for the given program when executed in a state s_0 where x has the value **3**. Determine a state s such that the derivation sequence obtained for the given statement and s is infinite. Identify what is the resulting value stored in variable y for given value of x. Use Table 2.

Question 2

[12 p.] Consider the statements below. For each statement determine whether or not it always terminates and whether or not it always loops. Try to argue for your answers using the axioms and rules of Table 1.

1. `while` $\neg(x = 1)$ `do` $(y := y * x; x := x - 1)$
2. `while` $(1 \leq x)$ `do` $(y := y * x; x := x - 1)$
3. `while true do skip`

Question 3

[10 p.] Which of the following statements are correct and which are incorrect? Choose correct answer. There is only one correct answer for each question.

1. The following statements have the same behavior:

   1. `if` $\neg b$ `then` S_0 `else` S_1
   2. `if` b `then` S_1 `else` S_0

   a) true b) false

2. The following statements have the same behavior:

   1. $x := y + 1; y := x;$
   2. $y := x; x := y + 1$

   a) true b) false

3. The following statements have the same derivation sequence (in structural operational semantics):

   1. `if` b `then` S_1 `else` S_2;
   2. `if` $\neg b$ `then` S_2 `else` S_1;

   a) true b) false

4. If $\langle S, s\rangle \Rightarrow^n \langle S, s\rangle$, then $n = 0$.

   a) true b) false

5. The following equality holds:

$$\mathscr{S}_{\mathrm{ds}}[\![\, S_1; (S_2; S_3)\,]\!] = \mathscr{S}_{\mathrm{ds}}[\![\, (S_2; S_1); S_3\,]\!]$$

   a) true b) false

6. The following equality holds:

$$\mathscr{S}_{\mathrm{ds}}[\![\, (\texttt{if } (x \le y) \texttt{ then } z := x \texttt{ else } z := y); \texttt{skip}\,]\!]s = s\,[z \mapsto \min(sx, sy)]$$

   a) true b) false

7. The *(* choose correct answer *)* is used to evaluate arithmetic and Boolean expressions on Abstract Machine.
   a) memory state b) memory abstarction c) evaluation stack

8. In *(* choose correct answer *)*, we define a language by assigning a mathematical meaning to functions.
   a) natural semantics b) operational semantics c) denotational semantics

9. *(* choose correct answer *)* is a method which specifies the operation of the program one step at a time and we specify the entire transition from a configuration (a tuple of the form $\langle statement, state\rangle$) to a final value or store.
   a) natural semantics b) operational semantics c) denotational semantics

10. In category $\mathscr{C}$, a *(* choose correct answer *)* is a mapping between objects. Each one has a unique source object and target object in $Ob(\mathscr{C})$.
    a) functor b) morphism c) identity

Question 4

Complete the definitions:

1. **[1 p.]** In mathematics and computer science, ... is the technique of translating the evaluation of a function that takes multiple arguments into evaluating a sequence of functions, each with a single argument.

2. **[4 p.]** Define the semantic function $\mathscr{S}_{\text{os}}$ for structural operational semantics:

$$\mathscr{S}_{\text{os}} : \quad \dots \quad \to \quad \dots$$

$$\mathscr{S}_{\text{os}}[\![S]\!]s =$$

3. **[5 p.]** Extend then language *Jane* with the construct

$$\texttt{repeat}\ S\ \texttt{until}\ b$$

   and specify the denotational semantics for it. Provide only general definition (in the form of new semantic rule) with the appropriate conditions (and boundaries).
   Hint. The semantics for the `repeat`-construct is allowed to rely on the existence of a `while`-construct.

4. **[2 p.]** Semantic domain is generally defined as *(* complete the definition *)*

5. **[10 p.]** Given the domain specific language for controlling the robot in two-dimensional system with syntax for statements:

$$\begin{aligned} S ::= \ & \textbf{left} \mid \textbf{right} \mid \textbf{up} \mid \textbf{down} \mid \\ & \textbf{left}\ n \mid \textbf{right}\ n \mid \textbf{up}\ n \mid \textbf{down}\ n \mid \\ & \textbf{skip} \mid \textbf{reset} \mid S; S. \end{aligned}$$

   Formulate natural semantics, structural operational semantics and denotational semantics for the following statements

   - choose one statement for moving in concrete direction in one step (from the first line),
   - choose one statement for moving in concrete direction in n steps (from the second line),
   - statement **skip** (an empty statement).

   As semantic domain, consider a set $\textbf{Point} = \mathbb{Z} \times \mathbb{Z}$. Define a state.

Table 1: Rules for natural semantics

$$\langle x := e, s\rangle \to s[x \mapsto \mathscr{E}[\![e]\!]s] \qquad (1_{\text{ns}})$$

$$\langle \texttt{skip}, s\rangle \to s \qquad (2_{\text{ns}})$$

$$\frac{\langle S_1, s\rangle \to s', \qquad \langle S_2, s'\rangle \to s''}{\langle S_1; S_2, s\rangle \to s''} \qquad (3_{\text{ns}})$$

$$\frac{\langle S_1, s\rangle \to s', \quad \mathscr{B}[\![b]\!]s = \mathbf{tt}}{\langle \texttt{if}\ \ b\ \ \texttt{then}\ \ S_1\ \ \texttt{else}\ \ S_2,\ s\rangle \to s'} \qquad (4^{\mathbf{tt}}_{\text{ns}})$$

$$\frac{\langle S_2, s\rangle \to s', \quad \mathscr{B}[\![b]\!]s = \mathbf{ff}}{\langle \texttt{if}\ \ b\ \ \texttt{then}\ \ S_1\ \ \texttt{else}\ \ S_2,\ s\rangle \to s'} \qquad (4^{\mathbf{ff}}_{\text{ns}})$$

$$\frac{\langle S, s\rangle \to s', \langle \texttt{while}\ \ b\ \ \texttt{do}\ \ S, s'\rangle \to s'', \mathscr{B}[\![b]\!]s = \mathbf{tt}}{\langle \texttt{while}\ \ b\ \ \texttt{do}\ \ S,\ s\rangle \to s''} \qquad (5^{\mathbf{tt}}_{\text{ns}})$$

$$\frac{\mathscr{B}[\![b]\!]s = \mathbf{ff}}{\langle \texttt{while}\ \ b\ \ \texttt{do}\ \ S,\ s\rangle \to s} \qquad (5^{\mathbf{ff}}_{\text{ns}})$$

Table 2: Rules for structural operational semantics

$$\langle x := e, s\rangle \Rightarrow s[x \mapsto \mathscr{E}[\![e]\!]s] \qquad (1_{\text{os}})$$

$$\langle \texttt{skip}, s\rangle \Rightarrow s \qquad (2_{\text{os}})$$

$$\frac{\langle S_1, s\rangle \Rightarrow \langle S_1', s'\rangle}{\langle S_1; S_2, s\rangle \Rightarrow \langle S_1'; S_2, s'\rangle} \qquad (3^1_{\text{os}})$$

$$\frac{\langle S_1, s\rangle \Rightarrow s'}{\langle S_1; S_2, s\rangle \Rightarrow \langle S_2, s'\rangle} \qquad (3^2_{\text{os}})$$

$$\frac{\mathscr{B}[\![b]\!]s = \mathbf{tt}}{\langle \texttt{if}\ \ b\ \ \texttt{then}\ \ S_1\ \ \texttt{else}\ \ S_2, s\rangle \Rightarrow \langle S_1, s\rangle} \qquad (4^{\mathbf{tt}}_{\text{os}})$$

$$\frac{\mathscr{B}[\![b]\!]s = \mathbf{ff}}{\langle \texttt{if}\ \ b\ \ \texttt{then}\ \ S_1\ \ \texttt{else}\ \ S_2, s\rangle \Rightarrow \langle S_2, s\rangle} \qquad (4^{\mathbf{ff}}_{\text{os}})$$

$$\langle \texttt{while}\ \ b\ \ \texttt{do}\ \ S, s\rangle \Rightarrow \langle \texttt{if}\ \ b\ \ \texttt{then}\ \ (S; \texttt{while}\ \ b\ \ \texttt{do}\ \ S)\ \ \texttt{else}\ \ \texttt{skip}, s\rangle \qquad (5_{\text{os}})$$

Evaluation:

Points	Grade
56-51 p.	**1**
50-43 p.	**2**
42-28 p.	**3**
27-29 p.	**4**
28-0 p.	**5**